LANDMARK UNIVERSITY, OMU-ARAN

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THE PERFECT GAS.

The characteristic equation of state.

At temperatures that are considerably in excess of the critical temperature of a fluid, and also at very low pressures, the vapour of the fluid tends to obey the equation;

$\frac{pv}{T} = constant = R$. No gases in practice obey this law rigidly, but many gases tend towards it. An imaginary ideal gas which obey the law is called a perfect gas, and the equation , pv/T = R, is called the characteristic of state of a perfect gas. The constant R, is called the gas constant. The unit of R are N m/kg K or kJ/kg K. Each perfect gas has a different gas constant.

The characteristic equation is usually written;

   pv = RT……1

or for m kg. occupying V m^3,

pV = mRT ……………..2

Another form of the characteristic equation can be derived using the kilogram-mole as a unit. The kilogram-mole is defined as a quantity of gas equivalent to M kg. of the gas, where M is the molecular weight of the gas (e.g since the molecular weight of oxygen is 32, then 1 kg. mole of oxygen is equivalent 32 kg of oxygen). From the definition of kilogram-mole, for m kg of a gas we have,

                m = nM ………………3

(where n is the number of moles)

Note: Since the standard of mass is the kg. kilogram-mole will be written simply as mole. Substituting for m from equation 3 in equation 2 gives;

$$pV = nMRT \text{ or } MR = \frac{pV}{nT}.$$

Now Avogadro's hypothesis state that the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure. Therefore V/n is the same for all gases at the same value of p and T. That is, the quantity pV/nT is a constant for all gases. This constant is called the universal gas constant and is given the symbol, R_o.

i.e $\qquad MR = R_o = \frac{pV}{nT} \text{ or } pV = nR_oT$ ………………4

or since $MR = R_o$ then, $\quad R = \frac{R_o}{M}$ ……………..5

Experiment has shown that the volume of one mole of any perfect gas at 1 bar and 0^0C is approximately 22.71 m^3. Therefore from equation 4,

$$Ro = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15} = 8314.3 \text{ Nm/mole K.}$$

From equation 5 the gas constant for any gas can be found when the molecular weight is known, e,g for oxygen of molecular weight 32, the gas constant ,

$$R = \frac{R_o}{M} = \frac{8314}{32} = 259.8 \text{ N m/ kg K.}$$

Ex. 1. A vessel of volume 0.2 m^3 contain nitrogen at 1.013 bar and 15^0C. If 0.2 kg of nitrogen is now pumped into the vessel, calculate the new pressure when the vessel has returned to its initial temperature. The molecular weight of nitrogen is 28, and it may be assumed to be a perfect gas.

Solution: From equation 5,

Gas constant $R = R_o / M = 8314/ 28 = 296.9$ Nm/kg K,

From equation 2, for the initial conditions,

$\qquad p_1V_1 = m_1RT_1.$

$\qquad \therefore m_1 = p1V_1 / RT_1 = \frac{1.013 \times 10^5 \times 0.2}{296.9 \times 288} = 0.237$ kg. Where $T_1 = 15 + 273 = 288$ K.

0.2 kg of nitrogen are added hence, m2 = 0.2 + 0.237 = 0.437 kg. Then from equation 2, for the final conditions,

$$P_2V_2 = m_2RT_2.$$

But $V_2 = V_1$ and $T_2 = T_1$,

$$\therefore \ p_2 = \frac{m_2 R T_2}{V_2} = \frac{0.437 \times 296.9 \times 288}{0.2} = \ \text{i.e } p_2 = \frac{0.437 \times 296.9 \times 288}{10^5 \times 0.2} = 1.87 \text{ bar.}$$

Ex.2.

0.01 kg of a certain perfect gas occupies a volume of 0.003 m3 at a pressure of 7 bar and a temperature of 131^0C. Calculate the molecular weight of the gas. When the gas is allowed to expand until the pressure is 1 bar the final volume is 0.02 m3. Calculate the final temperature.

Solution: From equation 2,

$p_1 V_1 = mRT_1$,

$$\therefore \qquad R = \frac{p_1 V_1}{m T_1} \ = \ \frac{7 \times 10^5 \times 0.003}{0.01 \times 404} = 520 \text{ N m/kg K. Where T1} = 131 + 273 =$$

404 K.

Then from equation 5,

$R = \frac{R_o}{M}, \quad \therefore \ M = \frac{R_o}{R} = \frac{8314}{520} = 16.$ i.e Molecular weight = 16. From equation 2,

$$p_2 V_2 = mRT2,$$

$$\therefore \ T_2 = \frac{p_2 V_2}{mR} = \frac{1 \times 10^5 \times 0.02}{0.01 \times 520} = 384.5 \text{ K}$$

i.e, Final temperature = $384.5 - 273 = 111.5^0$C.

3. A non-reversible process occurs for which pressure and volume are correlated by the expression $P = (V^2 + \frac{6}{V})$ where P is in bar and V is in m^3. What amount of work will be done when volume changes from 2 to 4 m^3?

$$P = \left(v^2 + \frac{6}{v}\right) bar, = \left(v^2 + \frac{6}{v}\right) \times 10^5 \ N/m^2$$

Work done, $w_{1-2} = \int P \partial v = \int_{v_1}^{v_2} \left(v^2 + \frac{6}{v}\right) \times 10^5 \ \partial v.$

$$= 10^5 \left[\frac{v^3}{3} + 6 log_e v\right]_2^4 = 10^5 \left[\frac{4^3 - 2^3}{3} + 6 log_e \frac{4}{2}\right]$$

$$= (18.67 + 4.16) \times 10^5 = 22.83 \times 10^5.$$

The Carnot Cycle
On studying heat engines and thermal machines, one is faced with a question very relevant: Given two sources of thermal energy at two different temperatures, one at a high temperature T_H and the other at a low temperature T_L, what is the maximum conversion of heat drawn from the source at high temperature that can be converted into useful work in an ideal heat engine (reversible one) that operates continuously in a closed thermodynamic cycle? First,

the Kelvin-Planck statement of the Second Law of Thermodynamics tells us that it is impossible to have a heat engine that will convert all the heat received from the high temperature source, Q_H, into useful work in a thermodynamic cycle. It is necessary to reject part of the received heat to the low temperature source, Q_L. In other words: it is impossible to have a 100% efficiency heat engine. A schematic of an operating heat engine according to Kelvin-Planck is shown in Fig. 3a. Second, Carnot devised that the heat engine that can achieve the maximum efficiency in continuously converting heat into work operating between the two heat sources is the one made up of four reversible processes as illustrated in the temperature-entropy diagram in Fig. 3b, which are:

(a) process 1–2—temperature raise from T_L to T_H in an adiabatic reversible process (isentropic);

(b) process 2–3—heat addition, Q_H, in an isothermic reversible process at T_H;

(c) process 3–4—temperature decrease from T_H to T_L in an adiabatic reversible process (isentropic);

(d) process 4–1—heat rejection, Q_L, in an isothermic reversible process at T_L. The thermal efficiency of any power cycle, gth; is the ratio of the network, W, and the heat received, Q_H.

$$\eta_{th} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} \quad \ldots\ldots 1$$

Where, the First Law has also been used, i.e., $W = Q_H - Q_L$.

From the T-S in diagram Fig. 3b, it is possible to notice that both heat addition and rejected are associated with entropy variation, i. e.

$$Q_H = T_H \Delta S \quad \ldots\ldots 2$$

and

$$Q_L = T_L \Delta S \quad \ldots\ldots 3$$

Therefore, substituting equations 2 & 3 into Eq. 1, one obtains the final form of the Carnot efficiency, gC; which is:

$$\eta_C = 1 - \frac{T_L}{T_H} \quad \ldots\ldots 4$$

This remarkable result shows that the maximum conversion of heat into work in heat engine operating continuously between two heat sources is limited by the ratio between the two heat sources temperatures. The lower the temperature ratio, the higher the Carnot efficiency. As a final remark, no 100% conversion can take place because it would require either a 0 K low temperature source, or an extremely high temperature source (mathematically, an infinite one), or both.
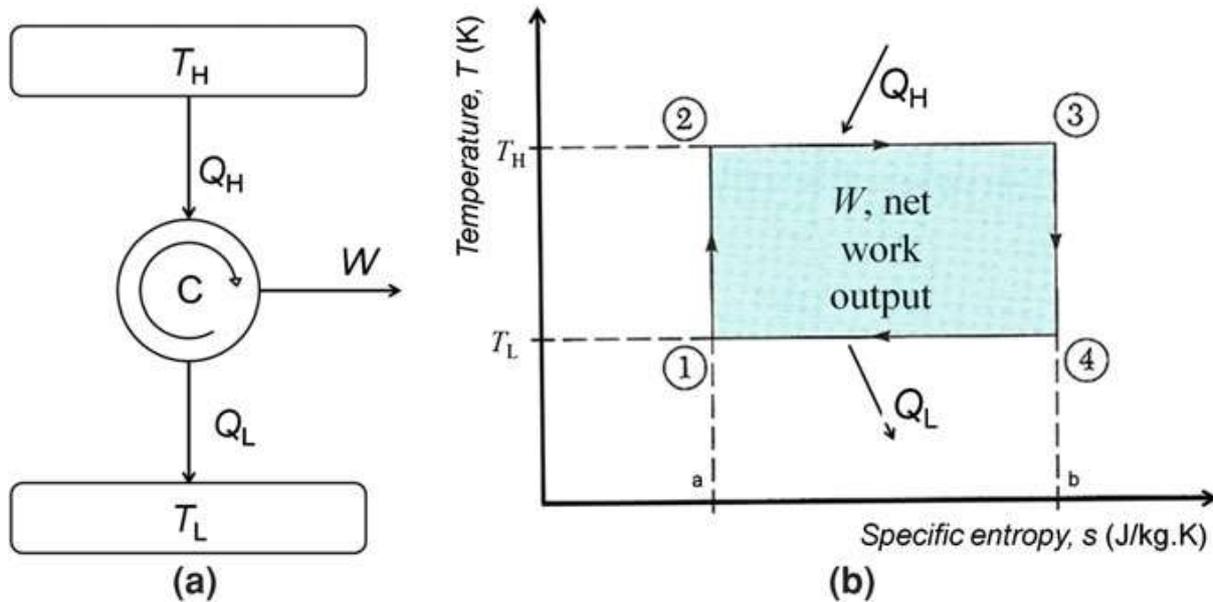
Fig. 3 a Schematics of a heat engine; b T-s diagram for a Carnot cycle.

Diesel Cycle

Diesel is the air standard thermodynamic cycle used in many internal combustion engines of many small to medium thermal power plants. The working principle of an internal combustion engine is somewhat different from a closed thermodynamic cycle as it occurs also with the Brayton cycle. Working fluid composition changes from plain air to combustion products and combustion and exhaustion processes are replaced by heat transfer processes. Therefore, there is an air standard cycle that reproduces the actual machine in order to capture its main features, such as the thermal efficiency.
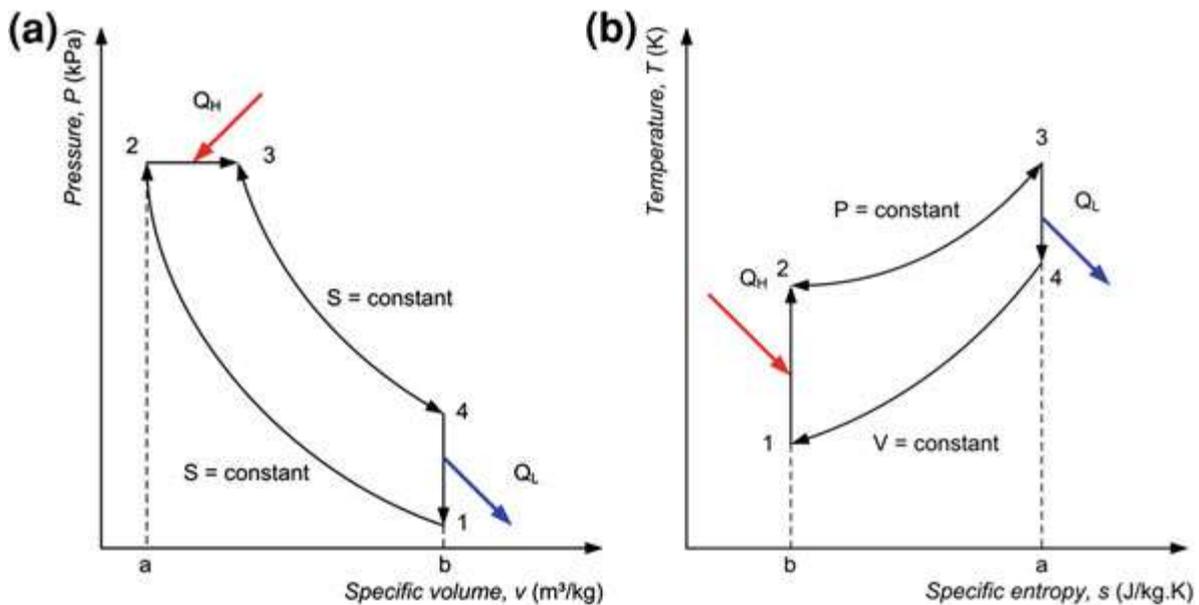

Fig. 2 Diesel cycle thermodynamic diagrams. a Pressure–volume diagram; b temperature entropy Diagram

Figure 2 shows the two relevant diagrams for Diesel Cycle analysis.

In Fig. 2a it is seen the pressure-specific volume diagram, while in Fig. 2b it can be seen the temperature-specific entropy diagram. The four ideal processes in a Diesel cycle are:

(1) process 1–2—isentropic compression, w_{comp} in the air standard cycle. Air is compressed from pressure P_1 to maximum pressure P_2. In turbocharged engines, P_1 is higher than the atmospheric pressure. In naturally aspirated engines, P_1 is the atmospheric pressure.

(2) process 2–3—heat addition, q_H, at constant pressure, $P_2 = P_3$, takes place in the air standard cycle. In actual engine, fuel is sprayed into the compressed air as its combustion takes place generating heat.

(3) process 3–4—in the air standard cycle compressed air at an initial high pressure and temperature T_3 undergoes an isentropic expansion, w_{exp}. In the actual engine, combustion products expand form high pressure P_3 to pressure P_4 generating shaft power.

(4) process 4–1—heat rejection, q_L, at constant volume, $V_4 = V_1$, occurs in the air standard cycle. In actual engine, the combustion products exhaust to atmosphere. Considering the ideal processes in Fig. 2, the following energy balances can be drawn.

$$\text{heat addition}: q_H = h_3 - h_2 = C_P(T_3 - T_2) \quad \text{-----5}$$
$$\text{heat rejection}: q_L = u_4 - u_1 = C_P(T_4 - T_1) \quad \text{.......6}$$
$$\text{compression work}: w_{comp} = u_2 - u_1 = C_P(T_2 - T_1) \quad \text{.......7}$$
$$\text{expansion work}: w_{exp} = u_3 - u_4 = C_P(T_3 - T_4) \quad \text{.......8}$$
$$\text{cycle net work}: w = w_{exp} - w_{comp} \quad \text{......9}$$

Thermal efficiency, η_{th}; of a cycle is defined as the ratio between the cycle net work and the heat added, i.e.:

$$\eta_{th} = \frac{W}{q_H} \quad \text{......10}$$

By substituting Eqs. 5, 6 and 9 along specific heats ratio, it yields;

$$\eta_{th} = 1 - \frac{q_L}{q_H} = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{\gamma(T_3 - T_2)} \quad \text{...... 11}$$

As the process 1–2 is isentropic, then

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = T_1 r_v^{\gamma - 1} \quad \text{.......12}$$

Where, r_v is the compression ratio. In Diesel cycle fuel is injected into the combustion chamber up to a certain point known as the cutoff ratio defined by

$$r_C = \frac{V_3}{V_2} \quad \text{......13}$$

Also, after a few manipulations it is possible to relate T_4 with T_1, which is
$$r_C = T_1 r_C^{\gamma} \quad \text{.......14}$$

Also, considering the process 2–3 is an isobaric one, then
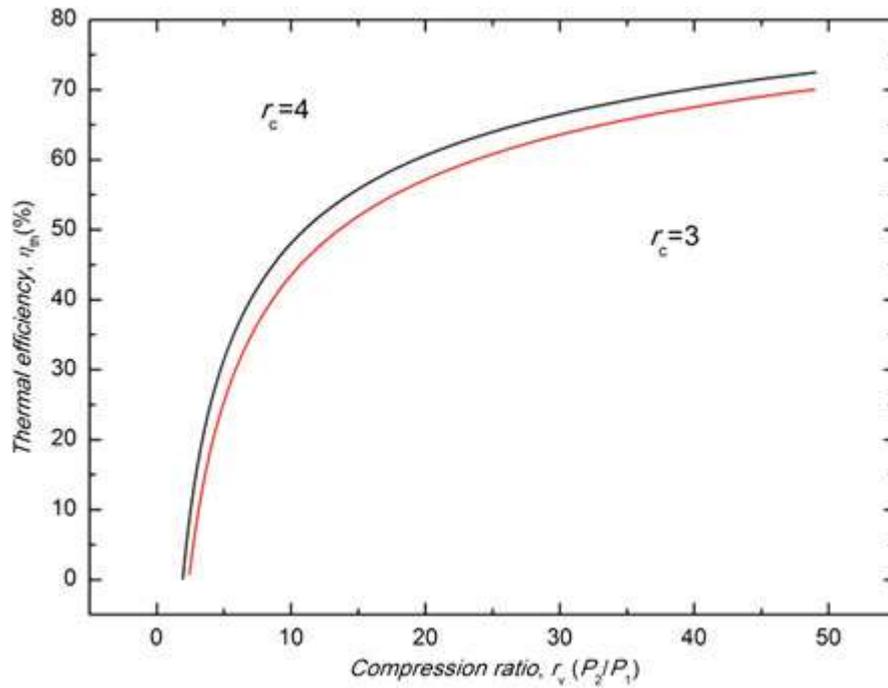
$$T_3 = T_2 r_C \quad \text{.......15}$$

Fig. 3 Diesel cycle efficiency as a function of the compression ratio, r_v, and cutoff ratio, r_c

By substituting Eqs. 12 through 15 into Eq. 11, one obtains

$$\eta_{th} = 1 - \frac{1}{r_v^{\gamma-1}}\left[\frac{r_c^{\gamma}-1}{\gamma(r_c-1)}\right] \qquad \ldots\ldots 16$$

Figure 3 shows the Diesel cycle efficiency as a function of the compression ratio, r_v, and cutoff ratio, r_c.